The life of Gotthold Ferdinand Eisenstein

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Gotthold Ferdinand Eisenstein (1823-1852) was one of the brilliant mathematicians of the nineteenth century. The main goal of this article is to give a translation – from German into English – of a Curriculum Vita that he wrote at age twenty, as part of his application for the Examination of General Maturity for University Entrance (Abitur). This has often been referred to as his autobiography. Some additional biographical information is also given, with the aim of providing some further insights into his life and character in general.

1 Introduction

Gotthold Ferdinand Eisenstein was born on April 16, 1823 and died on October 11, 1852, aged 29. He was, until his death, the only surviving child of Johann Konstantin (1791-1875) and his wife Helene (1799-1876). He lost five siblings to meningitis, the disease which became his fate too. His life is the story of a rather lonely, sad person, who was truly happy only when he could let his genius take its course. That genius led him to achieve in 29 years, what other persons would have needed their whole life for. During his first year at university he published papers almost on a weekly basis. The 27th edition of Crelle's Journal of 1844 contained 27 mathematical papers, 16 of those by the first year student G. Eisenstein [Rudio (1895)]. As a second year student, he was awarded an honorary doctorate. Two years later he received his habilitation, a prerequisite for lecturing at a German university. Such a qualification was normally granted after a minimum of three years and exceptions were extremely rare at that time.

But he had a price to pay. As a child he had hardly any friends. Although he was successful career-wise, his financial situation was always critical. During his time at the university he survived on short-term grants from the King and the Ministry of Education, which he owed solely to the intervention of Gauss and Alexander von Humboldt, two very important advocates of Eisenstein. Indeed, it was Humboldt who requested that Eisenstein be exempted from military service.

In many ways, Eisenstein lived in a world of his own. He was often misunderstood, even by his family. Because of his ill health he was always weak, shy and displayed a level of hypochondria – all of which led to him being subjected to harassment and teasing. Eisenstein just wanted to pursue mathematics, but his naiveté once even led to an accusation of plagiarism by Jacobi.

In a collection of letters from Eisenstein to M.A. Stern and to Gauss [Chelsea, *Mathematische Werke* (1975)], in addition to the usual interchange of mathematical ideas, Eisenstein reveals a great deal about his feelings at the time and his character in general. Paragraph 5 is about these letters.

A brief summary of Eisenstein's academic career and a figure showing mathematicians he was influenced by and whom he influenced is also provided.

Mathematische Werke – Gotthold Eisenstein by Chelsea Publishing (1975) served as the main source.

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"His [Eisenstein's] *knowledge of mathematics goes far beyond the scope of a Gymnasium's curriculum. His talent and zeal lead one to expect that some day he will make a considerable contribution to the development and expansion of science."*
(Schellbach, teacher at Friedrich-Wilhelm Gymnasium, Berlin 1843)
[Chelsea, *Mathematische Werke* (1975)]

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"These papers contain so much excellence and quality, that the author [Eisenstein] *will have a secured place of honor beside his predecessors, to whose work this publication follows with dignity"*
(Gauss, in a preface for Eisenstein's *Mathemathische Abhandlungen*, Berlin 1847)
[Chelsea, *Mathematische Werke* (1975)]

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2 Translation

2.1 Translator's Preface

The task of translating a 160-year-old German text requires more than the mere substitution of contemporary expressions for antiquated German ones. Extreme care must be taken to preserve the writer's original meaning and style. The translator is therefore constantly challenged to find the right balance between paraphrase and word-for-word translation.

How much more so with this particular text, which, although the work of a mathematician, is a text about life, pain, passion and emotions. Normally, when translating mathematical papers, it is often possible, even advisable to simplify or abandon distracting phrases in order to focus on the actual work and theorems. However, this is simply not possible with this text where the language chosen is part of the work. In fact, while reading and analysing the original German, it seemed to me as if Eisenstein had taken as much care in writing down his thoughts as I was taking to translate them. He was obviously very concerned about what other people thought of him, and yet he wrote about very intimate matters in an honest and revealing way. I have tried to preserve this sensitivity in my translation, while at the same time making the text accessible to a larger audience.

Although entitled, 'Curriculum Vitae', the text would nowadays be regarded as an autobiographical statement. It was submitted as part of an application for the university entrance examination when Eisenstein was 20 years old.

Although this paper does not contain any new discoveries and is biographical in nature, I hope many will benefit from this insight into the heart and mind of one of the world's relatively unknown geniuses.

2.2 Curriculum Vitae of Gotth. Ferdinand Eisenstein
[translated from: Chelsea, *Mathematische Werke* (1975)]

Before submitting my Curriculum Vitae to the laudable examination board I would like to take the liberty of first mentioning a few words about the direction of this work.

An overview of one's own life should not be composed merely of formalities, of descriptions of happenings and events that have affected oneself, of the circumstances in which one has lived, or of the experiences that one has had, from birth down to the present day; rather it must show the process of intellectual maturing; it must show how one has developed the mind and heart – little by little, through mistakes and errors – to reach a perspective from which one can look back on one's past life. The former merely provide the structure, while the latter provides the proper content. It is undoubtedly less challenging to merely list the historical facts of one's life (since this requires a long memory only) than it

is to think back – from the present perspective – to different stages of one's development, and to think and feel for a moment how one has once thought and felt as a child. However, I do not want to avoid the difficulty of this task and, in order that this work might serve its designation and purpose, namely, to show the degree of my moral and intellectual maturity, I want to try to combine a superficial description of my life with a background of my personal developments.

But before I think back to the past, I will first have a look at the present. This seems to be an appropriate point in time for this work; a precious resting place has been given to me from which to look back on my past life. I am 20 years of age – an important and significant age for a human being. At 20, one enters a completely new stage of life; one matures from lad to man, one moves from a period of education and dependence to a period of maturity and independence. Until now, being a child, other people, such as parents, friends and teachers, have taken care of one; one has been guided by others, and confined to a small sphere of activity only. Now, I have reached an age where I shall enter the real life, where, as my friends and family point out, independence and fortitude shall help me to act and live without assistance. Until now, one was still growing into manhood; now, people expect the firm character and mature attitudes which will be one's guides throughout one's life. At the age of 20, one can already see which path to follow; one can clearly see which expectations and aspirations to await. The child who was an object of mercy, whom one would forgive occasional lapses because of his youth, is now confronted with severe judgements about every step for which he is responsible, and which he must justify. If one has as yet been in doubt about one's vocation, now is the time to decide which profession to follow; now is the time to choose that certain path, which, with help of God, one intends to take for the rest of one's life. Hitherto, one has experienced a general preparation that should form a solid base for all possible directions. Now, one has to turn, with diligence and enthusiasm, to the acquisition of that knowledge and those aids which are necessary for the study of one's chosen subject. The twentieth year is therefore of great importance. It is, so to speak, the entrance gate which shall lead one from schooling to a life of work and productivity. For me, this moment is particularly critical. I am inspired by passionate love for a special science and wish to dedicate my life to the study of it. Up to now, I have been encouraged – with good reason – by parents and teachers to advance in all branches of science. Thus, no matter the force of the attraction, I have had to continuously fight my predominant affection. So shall it now be. Now my beloved desire will guide me through the rest of my life. If my present intention does not fail, I will no longer be urged to diffuse my strength in this way, but I will use it entirely for this one goal, which has been pointed out to me by an inner calling.

I shall end here with a description of my past life that I might look forward with peace, a fresh courage and trust in God.

I first saw the light of day on April sixteenth, in the year 1823 in Berlin. My parents were married in June of the preceding year and so I was the first son of their marriage. Later, I had another three brothers and two sisters – my beloved siblings – who, one after another, were taken by merciless death. So am I the only one of six children who is still alive. I was baptised by garrison preacher Ziehe, who also solemnised my parents' marriage. Thus, I became part of the Protestant Christians.

I count the first period of my life until the eleventh year, because at the age of 11 I left my parental home for the first time in order to go to boarding school in Dalldorf[1], two miles away from home. My time in Dalldorf caused me to remember with gratitude and emotion the sincere and true gentleness of my parents towards their first-born. For my part, I was very fond of my parents, particularly of my mother; I would cry if separated from her even for a moment. I did not find any joy in the company of children my age; and so, often I was alone, and spent a lot of time thinking alone. I only felt truly well and happy with my family. I avoided the loud and delightful play of other children of my age and I seldom got on well with any of them. But, if once I opened my heart to a friend, there remained a lasting, true and loving

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[1] The geographic scope of Berlin at the time when Eisenstein wrote his CV was quite different   than it is today. Most districts of Berlin today had not been part of the town in the early 19th century. Charlottenburg, nowadays part of the city-centre, is a famous example. Dalldorf has been renamed to Wittenau at the beginning of the 20th century and is today situated in the northern part of town.

friendship. I relied instead on the company of girls and grown-ups, particularly those with a serious nature; my greatest pleasure was to listen to the speech of a brilliant person. While other children were outside playing, I was sitting in the house listening to the conversation of grown-ups, or being taught or amused by my mother. It was she who gave me my first education, since my father was engrossed in his business, and could care for me very little. It was from my mother that I learned the first concepts, the first elements of knowledge. I remember how she interpreted the letters of the alphabet in a symbolic manner, in order to impress them on my knowledge: O was a gateway; a k was a key; thus every letter had some form associated with it.

In my youth I was extremely sickly and weak and therefore very irritable. Meningitis, which killed almost all of my siblings, had threatened my life as well. I recovered, but some bad after-effects remained, and they still seem to influence me today. These consequences are probably the reason for a recurrent mood of anxiety about my health, which has been pursuing me for the last two years and which I am not able to conquer.

In summer, while my father stayed in town to look after the business, my mother and I often resided in a house in an open part of town or beyond the gates. Thus, early on I became aware of the pleasures of life in the country, although even this delight, besides others, was tinged with bitterness, owing to my sickliness and irritability. My ill health had also caused some bad habits, which my mother had to contend with. For, no matter how intense her love towards me, she showed her true tenderness by not spoiling me at all and by no means indulging me.

As a result of my sickly state, I unfortunately fell behind many to whom I was mentally superior. I found it particularly hard to understand social and moral mores, so-called niceties. As a six-year-old lad I could easier understand a mathematical proof than I could understand why I should take my cap off in the parlour, or why I should use a knife to cut a slice of meat rather than tearing it apart with a fork. My refractoriness was the cause of a lot of trouble for my parents, since I always wanted to be told a reason for any action. With everything that depended upon intelligence and musing, I was at hand, but as soon as it was said, "That is a fact! This has to happen like that!" I became persistent and stubborn. With this mentality, where other children made good progress, I soon fell behind: that is in all those little skills and daily activities of life. Later, this gap caused me some embarrassment and unpleasantness and it required a lot of effort to make up for my inadequacy, when I could have acquired these skills earlier with ease. In general, it is a bad predicament for anyone to be in, who has not in his youth acquired abilities in these social proprieties, which we have common with animals. He will cause offence in many situations, for it is hard to attend to these trivial but unavoidable matters, especially when one's mind is already mature, if they are a habit developed early in life.

Among my parents' circle of acquaintances I mention one person, who should be well-known among important men. He already rests in peace, but, it was he who first saw and inspired my latent affection for mathematics. So far, this affection had come to light only in a desire to break things into pieces, and in the desire to discover what was concealed behind or, by crawling through hidden rooms and ways to find the secret of the ultimate door. Lautz it was who first introduced me to numbers and familiarised me with them, and although these impressions soon vanished due to my extreme youth, I owe him thankful memories. Lautz was a student of Pestalozzi and he introduced Pestalozzi's ideas to our family. Unfortunately, Lautz was a little bit eccentric. (Whether he was a genius or not, I do not know; a brilliant mind he certainly was.) He did not know how to comply with any circumstances, he was no friend of regular activities, neither was he able to submit his will to strangers. He resigned a marvellous position as one of the first lecturers at Cauersche Anstalt in Charlottenburg, which he had partly established and he finally died in misery. For my parents, he served as a warning beacon and they impressed his fate upon me after they had noticed my tendencies to follow a similar path.

From the age of six onwards I attended Bartelsche Schule in Scharenstrasse, which still exists today. There, the foundation of my scientific education was laid. During the first years I acquired a fundamental education: I still remember the torture of completing endless multiplication examples. From this, you might conclude, erroneously, that I lacked mathematical ability, merely because I showed little

inclination for calculating. In fact the mechanical, always repetitive nature of the procedures annoyed me, and indeed, I am still disgusted with calculations without any purpose, while if there was something new to discover, requiring thought and reasoning, I would spare no pains. I understood the form of grammar and languages with ease as I did everything that could be comprehended in a logical way.

Later, I learned history and Latin and made good progress with both. Besides other subjects, I was taught the fundamentals of physics. At that age there is usually no predominant inclination shown by a child. I was the same – an inspiring trigger from outside was missing, but as soon as it was present, that inclination erupted in full strength, as we shall see.

My parents had sent me to preacher Horn in Dalldorf in order that my health might improve in the country air.

This first separation from my family was extremely painful, and the separation from my parents was hardly bearable; often I would melt into tears for hours. I wrote most loving letters to my family, in which I called them all kinds of sweet names. In those days, owing to my fragile mood, I developed a certain poetic inclination. I wrote several poems about childlike love for parents. Later, my writing changed in favour of other themes and since then, if I am in the right mood, I write down a verse from time to time. I would like to take the liberty of attaching a later poem of mine[2] to this work, albeit that, I am convinced, it is poetically worthless.

At that time I appreciated agricultural work. I was given a piece of land, which I dug up by myself. I sowed seed and planted it and was filled with delight as flowers grew and blossomed.

My intellectual education was not well stimulated in Dalldorf, where I stayed for half a year; indeed, I forgot a lot of what I had learned in school, particularly Latin. However, I did receive my first lessons on the fortepiano by the sacristan of the village, Mr. Bergemann, whom I liked very much. I also did well in drawing, a talent which I later lost completely. After leaving Dalldorf, I never drew again and today, when I look at my drawings from that time, I must admit that I could not do half as well were I to try now. Music however, I still practise with passion and enjoyment, and I think I have developed some advanced skills on the fortepiano. I have even started composing; some of my compositions have at least pleased those to whom they were played. It is always good to develop a second talent besides one's profession, and one which will open one's way to society. In the end one cannot amuse people with mathematical theorems but everybody will listen with joy to a nice sonata or overture! During my stay in Liverpool and Dublin, where people were somewhat behind times, compared to the continent, I was considered a virtuoso, while here I am ranked merely as a piano-player of average ability.

I left Dalldorf to enter the Cauersche Anstalt in Charlottenburg, which, while falling into disuse after Cauer's death, had been re-established with the support of the government. I was one of the first to attend the new Institute, which is now led by director Mr. von der Lage. I could not agree at all with the manner in which I was treated here. The Institute was ruled by military discipline, which I could hardly endure, since, in my life till then, I had grown up with carefulness and tenderness. I had often been left to myself, and my actions were dependent on my mood and temper. Here, everything was arranged in a strict and unchangeable order. One had to get up, go to bed, work, play, all on command. We were under constant supervision by the teachers, and could not do anything at will. I spent many lonely, monotonous days under this unusually severe regime. And yet, I owe this Institute a debt. It was here that my mathematical talent first came to light. It was here that I first had the benefit of a systematic mathematical education. The method used by the director was as follows: each student had to prove the theorems consecutively. No lecture took place at all. No one was allowed to tell his solutions to anybody else and each student received the next theorem to prove, independent of the other students, as soon as he had proved the preceding one correctly, and as long as he had understood the reasoning. This was a completely new activity for me, and one which I grasped with incredible enthusiasm and an eagerness for knowledge. Already, with the first theorem, I was far ahead of the others, and while my peers were still struggling

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[2] Translator's note: I have been unable to locate this poem

with the eleventh or twelfth, I had already proved the hundredth. There was only one young fellow, now a medicine student, who could come close to me. While this method is very good, strengthening, as it does, the powers of deduction and encouraging autonomous thinking and competition among students, generally speaking, it can probably not be adapted. For as much as I can see its advantages, one must admit that it isolates a certain strength, and one does not obtain an overview of the whole subject, which can only be achieved by a good lecture. Once one has acquired a great variety of material through manifold and magnificent discoveries, one should absolutely be compelled to work among others: This is essential if one wants to understand – to a certain extent only – the essential and important aspects of this vast diversity of ideas. In the end, the best mathematical genius cannot discover alone what has been discovered by the collaboration of many outstanding minds and emerged from their common activities. For students, this method is practicable only if it deals with small fields of easily, understandable knowledge, especially geometric theorems, which do not require new insights and ideas. Thus, on leaving the Institute, the scope of my mathematical knowledge was rather limited in regards to quantity, despite my solid and proper understanding of the material taught. On entering the Gymnasium, I was, in fact, behind the other students. If I now look at the range of the subject of which I am master, it appears to me like a miracle, that I could achieve all this in as little as five to six years without any classes. But after I had acquired some necessary aids, I devoted myself with such passion and diligence to my studies, that it should be no surprise. I lived for my studies, and if I spent days and even half the night with my books, with an assigned task, or just working on my own ideas, then it was no strain at all, but the nicest comfort, provided that nobody disturbed or hindered me. If I then retired to rest, I slept with the pleasant anticipation of continuing my work the next morning and I would be back at my desk quite early.

Now I would like to return to my time in the Institute to show my progress in the other branches of science. I already knew the fundamentals of Latin grammar before coming to the Institute; there I read Cornelius Nepos and Julius Caesar; we attended exercitia[3] and had to sit extemporalia[4]. In history I was taught a general overview of the Roman and Greek era. Ancient Greek was not taught at all. I have lately studied the Buttmann grammar[5] by myself and got as far as the irregular verb forms. It was my wish to attend the tertia[6] of the Gymnasium[7], as soon as I came back to Berlin. I was not allowed to read Xenophon's Anabasis[8]; the books were taken away from me.

The teaching at the Institute took place in different classes, which were equivalent to grades below the tertia of the Gymnasium. I attended all the classes, my parents having decided they would leave me there until I had completely exhausted the available resources.

Life in the Institute was, as mentioned before, very uniform and strict. All of us slept in a big hall under a teacher's supervision. On his call we got up at 5:30 am, and washed and dressed to go down for communal breakfast, which consisted of milk and bread. Even at freezing temperatures the very spacious dormitory had not been heated, and so it often happened, that in winter, I had to use my boot jack to knock the ice off my stone water-mug to lure Neptune[9] from his crystalline dwelling. After breakfast we prepared the assigned tasks; lectures started at eight and lasted till four in the afternoon with a two-hour break for recovery and a midday meal in a big hall, which was attended by students and teachers alike. The hours until six were dedicated to study; the evening was free and we could use the time at will, albeit under a teacher's supervision. Where a teacher did not enjoy a proper authority, youthful *hubris* emerged

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[3] Latin, exercise

[4] Latin, unprepared: in this context usually a short unannounced test

[5] an ancient greek grammar, named after a German Philologist, Karl Phillip Buttmann
(1764-1829), often used as a grammar-textbook

[6] Tertia, Secunda, Prima: names for grades used in Gymnasia, Prima being the highest one

[7] name for secondary schools in Germany (still used today) where students achieve University entrance level after passing the *Abitur* (Examination of General Maturity) after 12th or 13th year of school attendance

[8] Xenophon was a pupil of Sokrates and involved in the march of the Athenians to Persia (401-399 B.C.), of which the story is about. The way this ancient war had been fought was probably not conform to the Prussian virtue.

[9] Roman God of the Sea – meaning in this context, that Eisenstein had to melt ice in order to get something to drink, because it was frozen

the stronger, the more so because of the strictness with which it had been suppressed during the remaining time.

On winter evenings the director was kind enough to teach me chess, which I enjoyed indeed. We were, by the way, almost always locked in our rooms or in the schoolyard, and rarely saw the town.

During that time my health was very poor, probably a consequence in part of this unusually harsh regime, which, while it might be quite advantageous for other young people, had the opposite effect on me. I suffered constantly from melancholy, I was never really merry, but rather earnest, I was often severely ill and I had to conquer fever and meningitis. Since I almost always felt sick, some thought that I was only pretending and so, in addition to my pain, I also had to endure insult. For the healthy and strong usually despise the weak and ill, and youth particularly so. Thus, early on I experienced what it means to suffer and to taste the bitter fruits of life; my very irritable mood meant that things that would hardly affect others, affected me twice as intensely. But I shall not grumble at this. I accept this pain with devotion, given as it is by the same loving hand of the Creator, who gave me a gift and love for the sciences.

In September 1837, after a long separation, I finally left Cauersche Anstalt and returned to my family, which at this time consisted of my parents, myself and a little sister, six years old, whom I loved sincerely, but who was soon wrenched away from me by death, by the will of God.

From that time on until July of the year 1842 I attended the Gymnasium in the capital, namely the Friedrich-Wilhelm Gymnasium, a glorious memorial to director Spilleke, where I completed the tertia and the sekunda, and then the Werder Gymnasium with director Bonnell, where I completed the prima. This period of my life was rather monotonous; I went to school every day, came home and completed my homework, and dedicated all the rest of my time to my mathematical studies.

My mathematical inclination became so dominant in this time that I neglected everything else and so it came about that my progress in other subjects was anything but superior. If my teachers were not too pleased with my overall achievements, I still managed to pass all my courses on time, and fulfilled the general requirements.

At this time I was already firmly determined to dedicate myself to the study of mathematics and did not question my career path, being guided by my inclination and my sense of inner calling. In this I am indeed more fortunate than many young men, since nature has destined me to follow a certain direction for my whole life – a direction I cannot change indeed, any deviation to different fields will be corrected by an inner force. I think it is sad when young men, who want to study, waver between this and that, now favouring this, then abandoning that, and find themselves still, in the last semester of the prima, not knowing what to study. Many only study as a result of false vanity, because they feel ashamed to take up a craft or any other profession, but at the end of the day, any position does a man credit, whose duty has been fulfilled with love and passion. Who does not perceive a true call and love for sciences, shall not study at a university: Even if this person would laboriously gather great knowledge, he would always remain a poor parrot, an un-sacred; he will never achieve independence, nor will he explore with enthusiasm's creative power the borders of science.

To explain the whole path of my mathematical efforts would be beyond the scope of this work. In general then, I offer the following only: The intense and sole appeal of mathematics, besides its content, has been for me the peculiar way of thinking required for dealing with mathematical problems. For me, the process of discovering and concluding new truths from known ones, the extraordinary clarity and evidence of theorems, and the ingenious ideas, which were foundations for a whole group of theories, all held an irresistible fascination. No other science had such a rich harvest to offer, and such an inexhaustible material with which to exercise mental capacities. Immanuel Kant mentioned clearly in his critiques that the field of mathematics was the only one where a human mind could hold forth without limits and *a priori* make constantly the most brilliant discoveries; constantly, indeed, because each new construct, if led by a brilliant mind, has to produce new results. Soon I became used to advancing deeper from single theorems into a wider context and understanding theories as a unity. Thus the idea of a mathematical

beauty dawned upon me. For mathematical beauty exists in the same way, that aesthetic beauty does. It is a beauty which can only be understood, if one is able, with full enthusiasm to see the complete picture – a whole system of discoveries, all following a main idea and leading to a final result, all linked together, in harmony and ingenuity, to form an organic entirety, just like a painting. There is also a mathematical rhythm or taste, which reads the analysis and then leads the approach and the development accordingly.

After I had acquired the fundamentals by private study (I never had a private tutor) I proceeded to advanced mathematics and studied, besides other books containing advanced material, the brilliant work of Euler and Lagrange about differential and integral calculus. I was able to commit this material securely to my memory and to master it entirely, because I made it a rule to compose every theory in writing as soon as I understood it. In addition, I tended to explore one theory in manifold ways. Often I would walk about the garden or in my room and demonstrate to myself – virtually teach myself – a whole series of theorems, of which I knew the proofs. I can recommend this method as very practicable. Lecturing was actually a passion of mine and I would have satisfied it amongst my peers with pleasure if they would not have confronted my enthusiasm with an icy coldness and if I would not have noticed with sadness how little interest there was among them for a science which I embraced with the deepest love imaginable.

As I was already well grounded in advanced calculus, my focus was turned, by different reflections, to number theory, which, strangely enough, I had previously considered fruitless. I now understood that I had been wrong, and dedicated myself from that time on, with great enthusiasm, to this branch of mathematics, whose treatment and content differs so completely from other branches. Indeed, it has its own independent existence – although, thanks to the cleverest and most exquisite parties of all branches now, the research of Dirichlet and Jacobi appear to join together. Since that time I have devoted myself to this twice-as-difficult and interesting branch of science, and I have made the most important aspects of it my own; even advancing to its latest discoveries. Number theory is less known among mathematicians owing to its peculiarity; but now it seems to occupy the most important place of all and to serve as a basis for all the latest research, as in Gauss' circle division and Dirichlet's theory of the number of quadratic forms. It is from this theory that I have chosen a subject as a presentation for the laudable examination board.

During the years 1840 to 1842 I attended Professor Ohm's lectures at the university with such energy and enthusiasm that I could hardly wait for the next class. When I had time, I also attended lectures given by the great Lejune Dirichlet, who unfortunately left us for a while in order to regain his strength in milder climates. I would gladly spend my whole life in his dignified praise, if I would not consider it arrogance to add my weak voice to the already existing general appreciation of magnificent minds. To cite Homer, even if I had a brazen heart and a thousand tongues, I could not describe the passion engendered in me by his magnificent and brilliant discoveries, not only in the one, but in all fields of Mathematics: how there was always a simple, beautiful and clear leading idea for the most complicated theories, an idea to which everything aligned, as to a central point; how he always knew how to find the key aspect, such that, no matter how one would distort the subject, one would always realise how it works.

The essential principle of the latest mathematic school, established by Gauss, Jacobi and Dirichlet, is in contrast to the past, such that it would enclose a problem as a whole by a brilliant method – with one leading idea – and would show the final result by a single strike in an elegant way, while previous methods tried to succeed by tedious, complicated calculations and deductions (as still used in Gauss' Disquisitiones). The old method proceeds from theorem to theorem, finally reaching a productive point, the other, presents a formula, which, right from the beginning, contains the full scope of truths of a whole complex. With the earlier method, theorems could also be proved, but now one can see a theory's true character, its actual inner mechanism. For example, Jacobi built the whole idea of inversion of elliptic integrals on the single idea that there exists every power of **x** except the term **1/x** in the differential of a two way power series[10]; Gauss built his magnificent idea of circle division, about which men had, since the time of Euclid despaired, on a peculiar order of integer numbers following the exponents of their congruent powers. But enough of that! The mathematician shall not pour his enthusiasm into words; that

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[10] nowadays called Laurent series

task is reserved for poets and artists, who can raise their art to heaven in melody and beautify their magnificence in brilliant paintings. With such a serious and noble science as mathematics, illusion shall not interfere; here only honest research and zealous striving are advisable. Every long-winded unscientific speaking about mathematical matters deviates from its true spirit: that blissful feeling, that can be felt only by those inspired by its truths and who can appreciate its true value, is delightful incense.

Having described my mathematical life and activities, a task which has taken almost all my time and absorbed all my energy, I have just a little to add. My reading has been very refined and was always chosen for me by my mother. Novels and pompous books, of the kind which young people usually devour, I did not like at all, and so my mind has been kept quite pure. I read the classics and educational books only, sometimes taking notes of those parts, which I was interested in, because they impressed me or because I thought they were important. Sometimes I also added a personal review as to how the book had affected me at the time. In this way I was able to observe how my taste and judgement changed or improved over time. I also kept a regular diary, in which I not only noted everything that happened but particularly how I had interpreted these occurrences and what I had thought and felt about them. Here, too, was evidence of how opinions waver and change during different periods of life.

I was taught truths of the Christian religion by Mr. Hossbach, who was councillor of the consistory. I enjoyed his teaching during the years 1839-40. At Easter in 1840, I was, by a solemn confirmation, admitted as a full member to those Christians who have vowed their confession in public and will now show their attitude and conduct, as Christ's true disciples, worthy of their Master and Saviour.

From this time until June of the preceding year, my life was routine, as life usually is during a continual stay in the same town or surroundings. Then, I left hometown and fatherland to explore new people, customs and a new, different life in foreign countries. Since my father had already moved his business to England two years previously, I travelled with my mother in the summer of last year to England and stayed in England, Ireland and Wales until two months ago. What I, a bookish fellow who was suddenly thrown into the wide world, have seen and experienced: how I woefully saw half of Hamburg in ashes, how I saw big cities with their peculiarities and humanity's inventive marvels, trains driving under rocks and foundations of houses, bridges under riverbeds, magnificent channels and harbours, proofs of the incredible things, which men, by combining forces, are capable of; how I travelled through beautiful scenery full of the charming fascination of nature; how I was touched by the majestic, endless expanse of sea; how I made six long sea journeys, one of which led us dangerously close to the cliffs of Anglesia under the suspension bridge under whose main arch there would be enough space for the whole of Berlin Castle and where boats could sail with upright masts; how I conquered Snowdon, England's highest mountain; how I made the acquaintance of important men, such as O. Connel; how I saw the saddest and worst poverty beside the most luxurious and richest opulence; how I saw people, to whom government, politics and liberty meant everything, people, who, in stark contrast to the superficiality and pride of other countries, appreciated only, what was true, dignified and useful; how I especially tried to understand the spirit of the universities there in order to compare it with the spirit of ours; how at last these manifold impressions came to influence the way I live, my judgements and my attitudes, so that I am now, after that journey, a completely different person. To explain and describe all this in succession and total clarity would alone require many pages, and would exceed the scope of this work by far. I would gladly submit a complete description of my journey – which I feel compelled to omit – in a separate document to the laudable examination board, if I had not the fear of boring them.

Enough! Although I had the intention to settle in Dublin and continue my studies there, my desire for the fatherland and my wish to serve it, as well as family affairs, have brought me back again. I have returned to concentrate with new enthusiasm and zeal on my studies, which I did not neglect at all on my journey. So, here I am, to pass, with help of God, the examination which will enable me to legally attend the university and to pursue a career in the Prussian state.

That is, then, a short overview of my past life. It is the life of a 20-year-old young man, who is still about to enter the real world and who cannot, like a mature man or like a wise old man, look back on proud and blessed achievements. It is the life of a man who is lacking in accomplishments, great deeds and

achievements, but not in everything good. He has determination, good intentions and the seed to grow good and beautiful things, although these have yet to spring up.

But also the way of perceiving this work and its judgements are those of a young man. If therefore, there is any one-sided description, any incorrect thought or false judgement, it is exactly this imperfection which makes this work what it is intended to be, namely an independent, objective description of my life composed from my present perspective. I therefore submit this youthful attempt at goodwill and indulgence to the laudable examination board, to the dear director and valued professors and teachers of the Gymnasium, and I dare to obediently recommend myself and my present wishes to your favour.
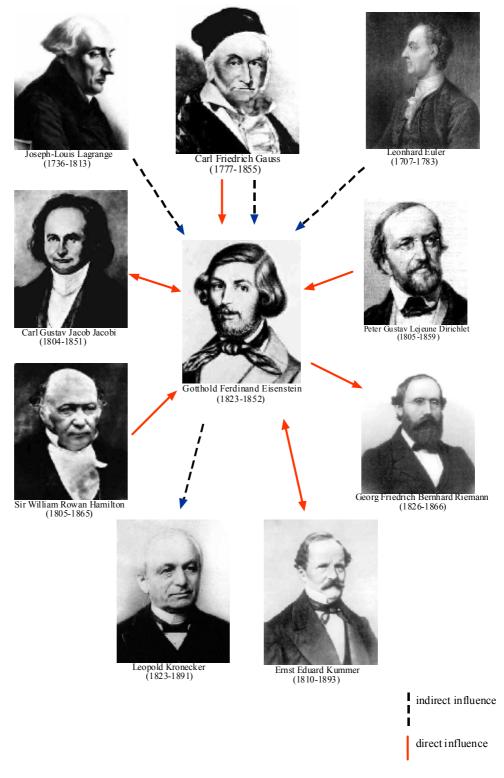
3 Eisenstein's Academic Career

[Timeline has been compiled from Chelsea, *Mathematische Werke* (1975)]

- 16.4.1823: Eisenstein was born in Berlin

- 1829 – 1833: Eisenstein attends a Primary School in Berlin

- 1833 – 1837: Boarding School in Charlottenburg – Cauersche Anstalt

- 1837 – 1842: Eisenstein is a pupil at Friedrich-Wilhelm Gymnasium (Tertia and Secunda) and then at Friedrich-Werder Gymnasium (Prima)
  During that period Eisenstein attends lectures from Dirichlet at the University of Berlin in leisure-time. By self-studying he masters Gauss' *Disquisitiones Arithmeticae* and papers and books of Euler and Lagrange. He starts to work on equations of third degree and elliptic functions.

- In spring 1843 he makes acquaintance of Hamilton in Dublin, who gives him a paper called *"On the Argument of Abel, respectly the impossibility of expressing a root of any general equation above the fourth degree"*. Hamilton asks Eisenstein to hand this paper to the Academy of Berlin. Back in Berlin Eisenstein hands in not only this paper but also one of his own papers *"Cubic forms with two variables"*.

- 22.9.1843: Eisenstein passes the *Examination of General Maturity for University Entrance* (Abitur)

- 21.10.1843: Enrolment at the University of Berlin

- January 1844: Crelle publishes the first papers of Eisenstein after recommendations by the Academy of Berlin. Crelle recommends Eisenstein to Alexander von Humboldt.

- March 1844: Humboldt invites Eisenstein and is from then on a powerful advocate for Eisenstein in both academic and financial regards.

- Easter 1844: Eisenstein meets Gauss in Goettingen, where he also meets Stern.

- 1844 Eisenstein's papers about *quadratic and cubic forms, reciprocity of cubic remainder, fundamental theorem for quadratic and biquadratic remainder, circle division, equations of third degree and elliptic and Abel transcendences* appear in Crelle's Journal.
- 15.2.1845: Eisenstein is awarded an honorary doctorate from the University of Breslau, initiated by Jacobi, Kummer and Fischer.

- 1846/1847: publications in Crelle's Journal focusing on elliptic functions

- 21.5.1847: Eisenstein's habilitation at the University of Berlin about *fundamental properties of rational functions*

- 1847 – 1852: Lectures and publications focusing on elliptic functions, number theory, higher reciprocity laws, cubic and quadratic forms.

- 24.4.1852: Eisenstein becomes a member of the Academy of Berlin

- 11.10.1852: Eisenstein died aged 29

4 Eisenstein's Mathematical Connections

Joseph-Louis Lagrange
(1736-1813)

Carl Friedrich Gauss
(1777-1855)

Leonhard Euler
(1707-1783)

Carl Gustav Jacob Jacobi
(1804-1851)

Peter Gustav Lejeune Dirichlet
(1805-1859)

Gotthold Ferdinand Eisenstein
(1823-1852)

Sir William Rowan Hamilton
(1805-1865)

Georg Friedrich Bernhard Riemann
(1826-1866)

Leopold Kronecker
(1823-1891)

Ernst Eduard Kummer
(1810-1893)

indirect influence

direct influence

[Pictures retrieved from Turnbull, 1998]

5 About the letters to Gauss and Stern
[Chelsea, Mathematische Werke (1975)]

Prof. Dr. M. A. Stern died on January 30, 1894 in Zurich, Switzerland. In his house in a small package one had found ten letters from Eisenstein with a sticker on it requesting to preserve these letters and if possible to publish them after his death. They had first been published by A. Hurwitz and F. Rudio in 1895. The letters to Gauss, preserved at the University library in Goettingen, had first been published by Chelsea Publishing in 1975.

In the chronologically first letter Eisenstein's to Stern, he asks him to give his kindest regards to everyone he (Eisenstein) knows in Goettingen, but *"You had better not greet Gauss from me, since to the good god, one can only pray and look up admirably"*.

The letters to Gauss are mainly mathematical. They would usually start with a submissive apology for writing him at all and stealing his time. Then he would immediately turn to his theorems and enthusiastically fill page after page. It seems like someone has finally found a person with whom he could adequately communicate about the things which count most for him. Nevertheless Eisenstein chooses his language carefully, always bearing in mind that he is writing to somebody he admires. A nowadays as exaggerated regarded kindness was not unusual in these times when writing letters to a person of higher standing. One letter starts with the words: *"The highest generosity and indulgence, with which Your Reverend Honor had the goodwill to accept my latest mathematical notes let me become so impudent, that I might dare to add the devoted request to…"*

The letters to Stern are also of mathematical nature, but are obviously the letters to a friend as well. Eisenstein and Stern would call each other *friends*. I am stressing the meaning of the word friend here because one can find a lot of references in these letters that would contradict the accepted meaning of friendship. Nine of the ten letters start with half a page full of apologies for not writing back for such a long time. Sometimes it took Eisenstein almost a year to answer a letter. He would attach not completed letters as a proof that he did indeed think of him but had nothing important to tell. Eisenstein refused to write *unmathematical* letters. *"Another letter, I thought later was too melancholic and unmathematical and had thus been dismissed"* he once wrote him.

Nonetheless, Stern was probably the only person Eisenstein could talk to about his personal feelings, health and concerns. But, as already mentioned, he wouldn't tell Stern even about his severest pain unless he could combine it with a new mathematical idea. This behavior made him end in isolation. He did not care much about anything but his work. Even his doctorate and habilitation he just mentions shortly in one letter as if they were just necessary evil for doing his work. His family he later called *people of money, who do not understand him*. His colleagues at the university accepted his mathematical work but abandoned him as a person. One letter says *"My relationship to the local scholars is as usual; approaching them would not be of use anyway, since their pride and aristocratic behavior to my person they would not give up. I am alone…"*

His only focusing on the theorems, one day caused a lot of trouble. Jacobi accused Eisenstein of plagiarism. It was about a priority claim. Eisenstein himself, as he writes in a letter to Stern was totally aware of the situation as he published the concerning paper and writes *"… responsible is only my naiveté, since I usually do not care about these superficialities, but think about the science only…"*

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